Influence modelling using bounded rationality in social networks

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***Abstract*—Influence models enable the modelling of the spread of ideas, opinions and behaviours in social networks. Bounded rationality in social network suggests that players make non optimum decisions due to the limitations of access to information. Based on the premise that adopting a state or an idea can be regarded as being 'rational', we propose an influence model based on the heterogeneous bounded rationality of players in a social network. We employ the quantal response equilibrium model to incorporate the bounded rationality in the context of social influence. The bounded rationality of following a seed or adopting the strategy of a seed would be negatively proportional to the distance from that node. This indicates that the closeness centrality would be the appropriate measure to place influencers in a social network. We argue that this model can be used in scenarios where there are multiple types of influencers and varying payoffs of adopting a state. We compare different seed placement mechanisms to compare and contrast the optimum method to minimise the existing social influence in a network when there are multiple and conflicting seeds. We ascertain that placing of opposing seeds according to a measure derived from a combination of the betweenness centrality values from the seeds and the closeness centrality of the network would provide the maximum negative influence.**

I. Introduction

Influence modelling in social networks is a key research problem with many applications over different domains. As a motivating example, consider the scenario where the present discussion on global-warming is operating in online social media and in social networks in general. With the issue of global warming, the actions of individuals, organisations and governments are deeply influenced by several key individuals who may be scientists, political figures and social figures. Thus, modelling the influence of such key players over the rest of the network would be an important research problem as it affects the spread of information over the network. This information spread may be key in determining the subsequent actions that would affect the resolution or the aggravation of the issue at hand.

Numerous attempts have been made to model the influence in a social context. Two classical models are linear threshold model and the independent cascade model [1]. Both these models take into account the neighbourhood effect of adopting a particular state by a node in the social network. Social influence modelling tries to address the optimisation problem of finding the optimum configuration of seeds to maximise the social influence. Under both these models, the optimisation problem of selecting the most influential nodes has been shown to be an NP-hard problem[1], [2]. Therefore, greedy algorithm is often used to come up with an approximated solution [2]. Another approach to model social influence has been to use the Page-rank algorithm based models, especially with respect to measuring the influence of micro blogs[3]. Game theoritic influential models too have been suggested to model social influence, where social influence is modelled as a strategic game[4], [5]. However, these models assume the prevalence of perfect rationality in players making their decision to adopt a particular state, even though in real-world players are boundedly rational [6]. In this work, we present a social influence model that is based on the bounded rationality of players in a social network. In the proposed model, the rationality of following an influencing node or adopting a strategy would be negatively proportional on the distance from the seed.

The rest of this paper is organised as follows. In the next section, we discuss the relevant background knowledge for our work. In particular, we discuss about the social networks, existing social network influence models and game theory. Then, we present our influence model based on bounded rationality and quantal response equilibrium model. Next, we simulate the propagation of social influence when seeds are placed at different configurations. We propose an efficient mechanism to find the optimum placement of seeds to counter the influence of existing seeds, when there are multiple types of contending seeds. Finally, we discuss our results and present our conclusions.

II. Background

In this section, we will discuss some of the key background knowledge that is essential for our discussion.

A. Social networks

A social network can be considered as a network structure that consists of social actors[7]. Analysis of social networks can be done based on different disciplines, such as psychology, sociology and statistics[7]. With the advent of complex network analysis[8], [9], [10], [11], there has been keen interest

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in using network analysis to study social networks as complex systems[9], [12]. In particular, the scale-free and small-world models that are prevalent in complex networks are prevalent in social networks[9].

In addition to the Scale-free networks, we would be using Erdos-Renyi (ER) random networks[9] and well-mixed networks in our study. The ER random networks are generated by randomly connecting links while in well-mixed networks all nodes are connected to each other. These models are used as reference models. These models help us to evaluate how the social influence model that we propose perform under varying topologies.

B. *Influence modelling in social networks*

Modelling of influence in social networks have gained much interest in the recent past. This is partly due to the potential that the emergence of online social networks present, in myriad of fields from online marketing of products to political campaigns[13], [14], [15], [16]. Especially due to advent of 'viral marketing' where word of mouth is used as a form of advertising through social media, the importance of social influence modelling has become even more prevalent[17], [14]. One key advantage in online social networks is it is possible to harness the meta information about the social network such as the underlying topology and the weights of the links, based on the data that is captured from the social interactions[14]. The key challenge in social influence modelling would be to identify the placement of 'seeds' or the influencing agents that would be able to create a cascading effect in the network, where the maximum possible number of nodes in the network are affected. This problem becomes even more complex when there are multiple types of contending seeds are in operation[18], [4]. Two main classes of influence or diffusion models are found in the literature, namely the linear threshold model and independent cascade model. Apart from that, recent interest has emerged on network topological influence models based on the Page-rank algorithm, and even based on game-theoretic models[3], [4]. Following is a brief introduction to some of the common social influence models found in the literature.

1) Linear threshold model: One of the most common models used to model social influence is the linear threshold model[1]. The assumption that is made in this model is that a node has a binary state of being active or inactive, with respect to a particular state that it is under influenced. Each node would have a random variable that dictates the fraction of nodes based on whose state which, it will switch or keep its current state. Formally put, each node v would have a threshold $\theta_v \epsilon [0, 1]$ that is randomly selected, which denotes the fraction of neighbours of node v that has to be active in order for node v to be active and vice-versa. Each node is affected by each neighbour w according to a weight $b_{v,w}$ such that, $\sum_{w \Diamond v} b_{v,w} \leq 1$, where we use the $\Diamond$ symbol to mean 'neighbour of'. A node is activated when the total weight of its active neighbours is at least θ_v:

$$\sum_{w \Diamond v} b_{v,w} \geq \theta_v$$

The random assignments of threshold θ_v account for the lack of knowledge of intrinsic latent tendencies of nodes to adopt neighbour strategies.

2) Independent Cascade model: In the independent cascade model [1], when a node v becomes active, it has a single chance of activating each currently inactive neighbour w. Each activation attempt would succeed with probability p_{vw}. Here too, the influence is defined in a binary fashion while multiple influence types are not considered. An extension for the independent cascade model has been proposed which allow the inclusion of negative opinions[18].

3) Page-Rank based influence models: Page rank algorithm was initially used by Google to rank the web pages based on rankings of their neighbourhood[19]. It can effectively be used to measure social influence, particularly in online social networks such as the blogsphere. Page-rank with prior has been suggested as one such possible influence model[20], which has been used to measure social influence in collaboration networks. It has also been used to evaluate microblog users' influence[3]. However, Page-rank is generally sought after to quantify the influence or the rank of each node rather than to identify to find the optimum seed arrangement to maximise or minimise social influence.

C. *Game theory*

In our influence model, we employ a game theoretic approach to quantify influence. Game theory is the science of strategic decision making[21], [22], [23], [24]. Different games such as the prisoner's dilemma game and the coordination game have been proposed to model different strategic decision making scenarios[21], [25], [26]. One of the pivotal concepts of Game theory is Nash equilibrium[27]. Nash equilibrium suggests that there exists one of more equilibria in strategic decision making scenarios, from which no player would benefit by deviating. One of the key assumptions in Nash equilibrium is that the players are fully rational[28]. In other words, it assumes that the players have full knowledge of the strategies and the payoffs of the opponents and would not have any cognitive or temporal limitation in calculating the optimum strategy[28]. The equivalent concept of Nash equilibrium in populations of players is the evolutionary stability of strategies[29], [30], [31]. If a strategy is evolutionarily stable, it would be able to wipe out any competing mutated strategies.

The assumption of full rationality does not hold for real-world players as they tend to have 'bounded rationality'. Bounded rationality is the non-optimal rationality of players due to the limitations of cognitive capacity, information availability or the computational time available for a player[6]. For this reason, the behaviour of real-world players may deviate substantially from Nash equilibrium[32].

1) Quantal response equilibrium: Quantal response equilibrium is a generalisation of Nash equilibrium, that accounts for the boundedly-rational or noisy agents. Probabilistic choice models are often used to incorporate stochastic elements in to the analysis of individual decisions. Quantal response equilibrium (QRE)[28] presents an analogous way to model games with noisy players. Probabilistic choice models such as logit and probit models are based on quantal response functions. They have the inherent feature where the deviations of optimal decisions are negatively correlated with the associated costs. Thus, in the QRE model, players are likely to select better choices than worse choices, although there is no guarantee that they will always select the best possible choice. Consider the payoff matrix given in Fig. 1, for a generic normal-form game. The logit function given in Eq.1 is often used to derive the equilibrium probabilities at QRE[28], [33].

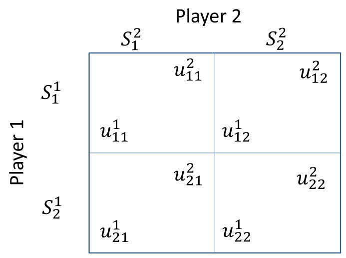

Fig. 1: The payoff matrix of a generic normal form game.

$$P_j^i = \frac{e^{\lambda_i Eu^i(s_j^i, P_j)}}{\sum_k e^{\lambda_i Eu^i(s_k^i, P_k)}} \tag{1}$$

Here, P_j^i is the probability of player i selecting the strategy j. $Eu^i(s_j^i, P_j)$ is the expected utility to player i in choosing strategy j, given that other players play according to the probability distribution P_j. The total number of strategies that player i can choose from is given by k.

QRE produces a mixed strategy equilibrium, where the choice probabilities give the equilibrium probabilities of a player with a particular value of bounded rationality. In the logit QRE function given in Eq.1, λ_i is known as the rationality parameter of player i. By varying it, it is possible to vary player i's ability to respond to the opponent's strategy distribution and the payoffs obtained under each strategy. Accordingly, rationality parameter can be regarded as a measure of a player i's rationality. It has been shown that as $\lambda_i \to \infty$, the equilibrium gets closer to the Nash equilibrium, and as $\lambda_i \to 0$, the player would operate in a totally random (thus irrational) fashion[28]. Within this range, the rationality parameter would produce equilibria of boundedly rational players. For instance, based on this property, a topological model of bounded rationality has been proposed[34]. Thus, we can use this QRE logit function to derive the equilibrium probability distributions of players who operate under non perfect rationality.

III. Modeling influence using bounded rationality

Based on the background theoretical knowledge, we present a social network influence model based on game theory and bounded rationality of nodes. We first model the social influence of nodes as an influence game, where there would be influencing nodes or 'seeds' and followers operating in a network of players. The seeds would continue to operate with a permanent binding to a particular state. This inclination may be due to some external knowledge or an incentive the seed may have from the external environment. In the context of a influencer-follower scenario, the bounded rationality of a follower would be a 'rationality of following'. Higher the rationality of a follower with respect to a seed, higher the probability of it following the state of the seed. In this social influence model, we assume that the rationality parameter of a particular follower is negatively proportional to its distance from the seed. This assumption would account for the random noise that would be accumulated as the followers move further from a seed. Based on the rationality of following, we can measure the probability of a follower being at the state of the influencing node or the seed node. Thus, our model does not produce a binary outcome where the followers would be active or inactive in a binary fashion, rather the result would be a probability on which a follower would adopt the probability of the influencer. In a game theoretic terminology, the follower's probability distribution would be a mixed strategy equilibrium, where the two strategies would be whether to adopt the strategy of the seed or not. Formally put, the follower probability $p_{n,s}$ of adopting the active state s of the seed would be,

$$P_{n,s} = \frac{e^{\beta_{n,i}.U_s.P_{i,s}}}{e^{\beta_{n,i}.U_s.P_{i,s}} + e^{\beta_{n,i}.U_{-s}.P_{i,-s}}} \tag{2}$$

where,

$P_{n,s}$ - Probability of the follower node n being at state s (active state)
$\beta_{n,i}$ - Following rationality of node n with respect to node i
U_s - Utility of adopting the state s
$P_{i,s}$ - Probability of the influencer i being in state s (this is always 1)
$P_{i,-s}$ - Probability of the influencer i being not in state s (this is always 0)
U_{-s} - Utility of not being in state s (inactive state)

We add random noise to the followers with the assumption that the rationality of a node of following the influencer is negatively proportional to the distance from the seed or the influencer. Thus, $\beta_{n,i} \alpha \frac{1}{d_{n,i}}$ where $d_{n,i}$ is the distance along the shortest path from the influencer i to node n along the shortest path. As the follower moves further from the seed, the rationality parameter reaches 0, making them behave randomly. If the followers are placed closer to the seed, then there would be higher rationality and thus a higher probability of following the state of the seed. Another important factor to note is the not only the distance from the seed, but also the reward or utility of adopting the state too play a significant role in determining whether a follower would adopt the state of the seed. We can extend the above model to a case where there are multiple seeds or influencers instead of a single influencing node.

$$P_{n,s} = \frac{\sum_{i=1}^{N} e^{\beta_{n,i}.U_s.P_{i,s}}}{\sum_{i=1}^{N} e^{\beta_{n,i}.U_s.P_{i,s}} + \sum_{i=1}^{N} e^{\beta_{n,i}.U_{-s}.P_{i,-s}}} \tag{3}$$

where N is the total number of influencers in the network. In the above model, each node would have a separate rationality parameter for each influencer, based on the distance to them. Thus, it would capture the varying network distances from each influencer to more accurately predict the status of the follower.

This would imply that in a population that is closely knitted would have a higher tendency of following a seed compared to a population that is sparsely connected. Further, small-world networks[9] would tend to leverage social influence as they have relatively low average path lengths[9].

IV. Modeling social influence under opposing influencers using bounded rationality

The model we introduced could be used to model the influence of a single type of influencers. Yet, in most real-world social influence scenarios, there would be conflicting interests at play. There would be influencers with a negative influence as well as positive influence on the same state. In addition to that, there could be instances where there are multiple influencers that are mutually exclusive from each other. A good example of this are political campaigns where there would be more than two candidates running. Thus, we can easily extend the above model to account for two opposing types of influencers of states S_1 and S_2 as given in Eq. 4.

Here,

P_{n,S_1} - Probability of the follower node n being at state S_1
$\beta_{n,i}$ - Following rationality of node n with respect to the influencer i
$\beta_{n,j}$ - Following rationality of node n with respect to the influencer j
U_{S_1} - Utility of adopting the state S_1
P_{i,S_1} - Probability of the influencer i being in state S_1 (this is always 1)
$P_{i,-S_1}$ - Probability of the influencer i not being in state S_1 (this is always 0)
P_{i,S_2} - Probability of the influencer j being in state S_2 (this is always 1)
$P_{i,-S_2}$ - Probability of the influencer j not being in state S_2 (this is always 0)
U_{-S_1} - Utility of not being in state S_1
U_{S_2} - Utility of adopting the state S_2
U_{-S_2} - Utility of not being in state S_2

The rationality parameters with respect to each influencer would again be dependent on the distance of the node in concern from each of the influencers. Note that the followers can take either of the two states S_1 or S_2 under the influence of the two types of influencers. However, it does not account for a neutral state where the followers may not follow either of the two types of influencers. If a neutral state is considered, then the numerator should only contain the exponent of S_1, as in that case a node being influenced to be in state $-S_2$ does not mean it would automatically adopt S_1. Further, it is possible to extend the same model to take into account multiple types of influencers and not just two opposing types, since every influencer state can be regarded as a possible strategy a follower could adopt with heterogeneous rationality levels.

V. Optimizing influence using bounded rationality models

In this section, we propose a method to place the influencers in order to maximise their influence on the population, based on the bounded rationality based influence models that we've proposed. We mainly look at two scenarios. One is where the network has only a single type of influencers and the requirement is to select the placement of influencers to miaximize their influence. This is termed as the influence maximisation problem in the literature[1]. The other is the scenario where the network two kinds of opposing influencers. Supposing the network is already occupied with one type of influencers, we need to identify the optimum way of placing the rivalling set of influencers, so that the influence of the originally placed influencers is minimised. These optimisations would be applicable to the influencer-follower game that we discussed earlier and also more general strategic decision making situations.

Firstly, lets consider a scenario where there is only one type of seeds or influencers in a network in a influencing game. Since the bounded rationality of following is inversely proportional to the distance of the followers from the influencing node. Thus, the influencers are best placed in a network where the distance to the followers is minimum. The natural candidate to locate that placement would be the closeness centrality of the network, since closeness centrality is used to identify the nodes that have average minimum shortest path distance to the other nodes within the network. The Eq.5 depicts the equation for calculating the closeness centrality of a node. When the influencers are placed according to the closeness centrality of network, the influence on the rest of the network would be highest, when there is only one type of positive influence at play.

$$C_H(x) = \sum_{y \neq x} \frac{1}{d(y,x)} \tag{5}$$

where $d(y,x)$ would be the distance along the shortest path from node x to node y.

Next we'll look at what would be the most optimum method to place the rivalling or conflicting influencers when there are multiple opposing influencers in operation. Assuming that the original influencer or influencers are already placed in a network, there would be two factors that affect the effectiveness of the rivalling influencers. Those two factors are,

> Active factor - The distance from the rivalling influencers to the follower nodes in the network
>
> Passive factor - The ability of the rivalling influencers to 'block' the influence of the original influencers

The first factor could be measured using the closeness centrality of the network. We call it an active factor since the oppposing nodes would directly influence the remaining nodes in the network to follow them. The second factor becomes relevant since the rationality is spread through the network topology. If a node is occupied by an opposing influencer, then

$$P_{n,S_1} = \frac{\sum_{i=1}^{N} e^{\beta_{n,i}.U_{S_1}.P_{i,S_1}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{-S_2}.P_{j,-S_2}}}{\sum_{i=1}^{N} e^{\beta_{n,i}.U_{S_1}.P_{i,S_1}} + \sum_{i=1}^{N} e^{\beta_{n,i}.U_{-S_1}.P_{i,-S_1}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{S_2}.P_{i,S_2}} + \sum_{j=1}^{M} e^{\beta_{n,j}.U_{-S_2}.P_{i,-S_2}}} \quad (4)$$

it no longer would take part in spreading the influence of the original influencers. From the perspective of the original set of influencers, the opposing influencers would cease to exist in the network, in their quest to spread their influence. Thus, simply by being placed in positions where the distance from the original influencers to the followers could be increased, the rivalling influencers can minimise the influence of the original set of influencers. In other words, the opponent influencers are best placed in 'between' the original influencers and the followers of the network. Thus, another well-known centrality measure, which is the betweenness centrality can be employed to identify the topological positions where the influence of the original influencers would be minimised. However, we need not calculate the betweenness cetrality values of all nodes in the network to identify these positions. Only the betweenness cetrality values for the original influencers would be sufficient to identify the topological positions where there influece can most effectively be interfered. We call this the passive factor of negative influence since the opponent influencers reduce the influence of the original influencers simply by passively occupying the high betweenness centrality nodes.

By combining these two active and passive negative factors, we can derive a measure that could be used to identify the positions where it would be most effective to place the negative influence seeds. Thus, we can propose an algorithm to optimally place the negative influencers in a network where there are already existing positive influencers. Algorithm 1 details out the steps to calculate the proposed combined measure. In step 1, the betweenness centrality values from the original seed set gives the positions where the influence of the original seed set would be hindered, had them been occupied by rivalling seeds. On the other hand, the closeness centrality of the entire network is measured so that the opposing seeds would be 'closest' to the other nodes distributed in the network. By combining these two measures, it is possible to come up with the optimum metric, according to which the opposing seeds may be distributed in the network.

Algorithm 1: Optimum influence arrangement of negative influencing seeds, when a network is already occupied with positive influencing seeds.

Data: Network topology, Seeds
Result: Optimum arrangement of opposing seeds

1 calculate the betweenness centrality values of the nodes from the original set of seeds;
2 calculate the closeness centrality of the entire network;
3 get the average measure of these two measures;
4 order them in descending order;
5 place the opposing seeds in the order of the combined measure;

A. Complexity analysis

Influence maximisation is an optimisation problem that is tries to optimally place the seeds in order to the influence spread. Under the independent cascade model and linear threshold model, the influence maximisation becomes an NP-hard problem[1]. Therefore, in order to solve them, it is necessary to employ a greedy algorithm to find the optimum seed arrangement. The greedy algorithm of optimising influence spread outperforms the degree and centrality based heuristics[1], [2]. Still, it requires an approximation using the Monte-Carlo simulations of the influence cascade model over a number of interactions to obtain an accurate estimation of the influence spread. Thus, it may be inefficient in a sufficiently large network with multiple seeds involved.

Using the proposed bounded rationality based influence model, we can propose an exact algorithm to identify the optimium set of influencers without depending on heuristics. The time complexity of finding the optimum placement of a single type of influencers would be $O(V, E)$ where V would be the number of vertices and E would be the number of edges in the network, which is the time complexity of calculating the closeness centrality. The time complexity of optimising the negative influence when there are opposing influencers would be $O(V, E) + O(V_S, E)$, where V_S would be the initial set of seed nodes. Based on the proposed influence model, it is possible to measure the actual optimisation problem in polynomial time. Also, instead of an approximation, it is possible to derive an exact solution using the bounded rationality based influence model.

VI. Methodology

In this section, we present the methodology used to perform simulations to demonstrate the applicability of the bounded rationality based influence model. First, we compare three different network topologies, scale-free, ER random and well-mixed, in how they would facilitate the influence spread under a bounded rationality influence model. In order to do that, we place a seed at the hub of each network and measure the average probability of following the seed, over the network. The network with the lowest diameter would be the one that facilitates most influence. The two states that we consider are either active or inactive states where active refers to adopting the state of the influencer. However, since the QRE model gives a probability distribution of whether a node would follow the influencer or not, the outcome would not be a binary state distribution, rather a probability distribution of being in the active state. The payoff of adopting the state of the seed is set to be 1 and not adopting the state is set to 0.

$$p_{n_1} = \frac{e^{(\beta_{n_i} \cdot p_i^1 . 1)}}{e^{(\beta_{n_i} \cdot p_i^1 . 1)} + e^{(\beta_{n_i} \cdot p_i^0 . 0)}} \quad (6)$$

Here p_{n_1} would be the probability of node n being in state 1, where state 1 denotes being active and 0 denotes being inactive. β_{n_i} would be node n's 'rationality of following' with respect to the influencer i, which would be dependent on node n's distance along the shortest path from the influencer i.

Therefore, $\beta_{n_i} = c/d_{n_i}$ where c is a constant and d_{n_i} is the distance of node n from node i along the shortest path. 1 and 0 are the payoffs of state 1 and 0 respectively.

Next, in order to test how the payoff of following affects the probability distribution of following the seed, we performed the same experiment while varying the payoff of the active state. A scale-free topology was used for this simulation and the variation of the average probability of following over the payoff of the active state was observed.

Then, we observed the optimum method to reduce the influence of the hub by placing opposing influencers in the network. To test this, we considered two different scenarios. In the first one, the hub is placed with the original influencing node and 4 opposing influencers are distributed according to different configurations. In the first configuration, they are placed in the remaining hubs in the order of the degree. In the next two configurations, the opponents are placed in the order of betweenness from the hub and the closeness centrality of the network, respectively. In the 4th configuration, the opponents are placed in the order of the combined measure that we presented in algorithm 1. We compared and contrasted the variation of negative influence by observing the adoption probability under these 4 configurations. The same experiment was then repeated for multiple conflicting influencers on both sides. The original set of influencers were randomly distributed and 4 opposing influencers are placed according to the four configurations mentioned above. This experiment was repeated over 20 iterations to account for the effect of randomness in the initial configuration of influencers. We assume that the followers would adopt either of the active or the negatively active states, thus they wouldn't be in an inactive state. It should be noted that an inactive state could also be incorporated by extending the QRE model that we use. The payoffs of following each type of influencers were set to 2 and 1 respectively. In each type of influencer, the payoff of not following the influencer would be a negative payoff of -2 and -1 respectively. Eq. 7 shows how the probability of a node being in state s, which is the state of the original set of influencers, being calculated using QRE and distance induced bounded rationality.

Here, p_{n_1} would be the probability of node n adopting state s. β_{n_i} would be the rationality of following the influencers of state s where N is the number of such influencers. Also, β_{n_j} would be the rationality of following the influencer of state $-s$ where M would be the number of such influencers. 2 and -2 are the payoffs of either following or not following the influencers of state 1. 1 and -1 are the payoffs of either following or not following the influencers of state $-s$. The rationalities of following would be inversely proportional to the distance from each influencer. For instance, the rationality β_{n_j} of following an influencer of type j is set as c/d_{n_j} where c is a constant and d_{n_j} is the distance from node n to influencer j.

These experiments enable us to evaluate how the bounded rationality based influence modelling can be applied in a influencer-follower scenario, where there are a dedicated set of influencers or seeds and the rest of the population is following them.

VII. Results

The table I shows the comparison of the average following probability under the three network topologies considered. As the table shows, the scale-free topology facilitates the highest average following probability compared to the ER random and lattice topologies. This is due to the fact that a scale-free network of a comparative size and average degree may have a lower diameter compared to a ER-random or a lattice network.

TABLE I: The average probability of following the seed in different topologies

Network topology	Average probability of following
Scale-free	0.95
ER Random	0.89
Lattice	0.51

The Fig. 2 depicts the variation of the following probabilities in a scale-free network, when the seed is placed at the hub and when the payoff for adopting the state of the seed is increased. As shown in the figure, there is a clear positive correlation between the payoff of the active state and the influence spread. Thus, this shows that not only the topology of the network and the positioning of the seed(s) but also the payoff of the active state is critical in determining the spread of influence.

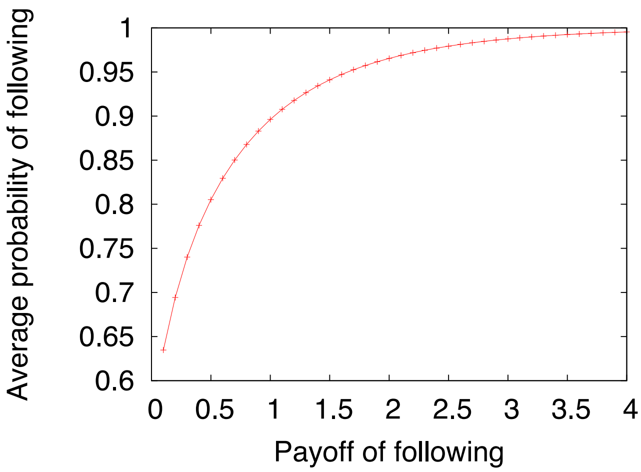

Fig. 2: The variation of the probability of following against the payoff of the active state.

Next, we depict the results when two conflicting types of influencers are placed in a scale-free network. The original influencer is placed in a hub and 4 opposing influencers are place according to 4 different configurations. Namely, they are placed according to the degree centrality of the network, betweenness centrality from the hub, closeness centrality of the network and a combination of the 2nd and 3rd measures (as discussed in algorithm1). Fig. 3 shows the comparison when the influencing seed is placed the hub in opposition to 4 counter influencers in the same 4 configurations. As the figure shows, the combined measure method proposed in algorithm 1 gives the best results in terms of maximising the negative influencer, thereby reducing the average probability of following. The table II shows the average probability of

$$p_{n_s} = \frac{\sum_{i=1}^{N} e^{(\beta_{n_i} \cdot p_i^s \cdot 2)} + \sum_{j=1}^{M} e^{(\beta_{n_j} \cdot p_j^s \cdot -1)}}{\sum_{i=1}^{N} e^{(\beta_{n_i} \cdot p_i^s \cdot 2)} + e^{(\beta_{n_i} \cdot p_i^{-s} \cdot -2)} + \sum_{j=1}^{M} e^{(\beta_{n_j} \cdot p_j^s \cdot -1)} + e^{(\beta_{n_j} \cdot p_j^{-s} \cdot 1)}} \tag{7}$$

following when the opposing seeds are placed in the 4 different configurations. The results reiterate that it is the negative seed placement method discussed in the algorithm 1 that provides the optimum reduction of the influence from the original seed.

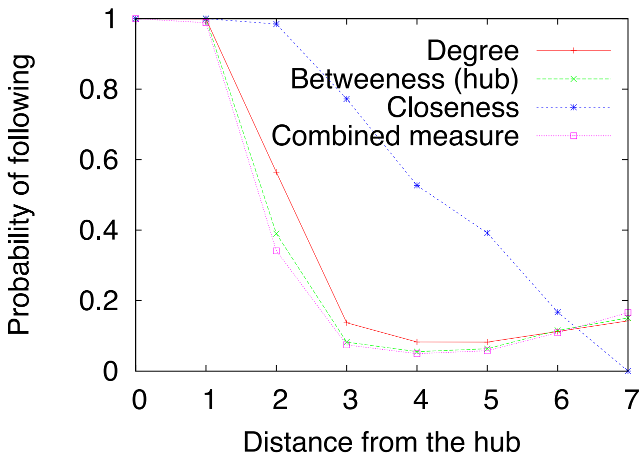

Fig. 3: The variation of the probability of following against the distance from seed placed at hub in a scale-free network. 4 opposing seeds are placed in the order of (i) degree, (ii) betweenness from the hub, (iii) closeness centrality, (iv) combination of both of 2 & 3.

TABLE II: The average probability of following the seed in different topologies. The opposing seeds are placed in 4 different configurations.

Opposing influencer configuration	Average following probability
Degree	0.29
Betweenness (hub)	0.21
Closeness	0.79
Combined measure (2 & 3)	0.19

Fig. 4 depicts the variation of following probability against the average distance from the influencing seeds when multiple original influencers are placed randomly. The results are averaged over 20 independent runs. As the figure depicts, it is the combined measure used to determine the placement of the opposing seeds that mitigate the influence of the original set of influencers, most effectively. This is further confirmed by the comparison of average probabilities of following, given in table III.

VIII. Conclusion and Future work

In this work, we propose a novel social influence model based on the bounded rationality of agents in a social network. First, we model the social influence as an influence game where there are two types of players, influencers and followers. The followers are assumed to be following the influencers based on a measure of bounded rationality, which is inversely

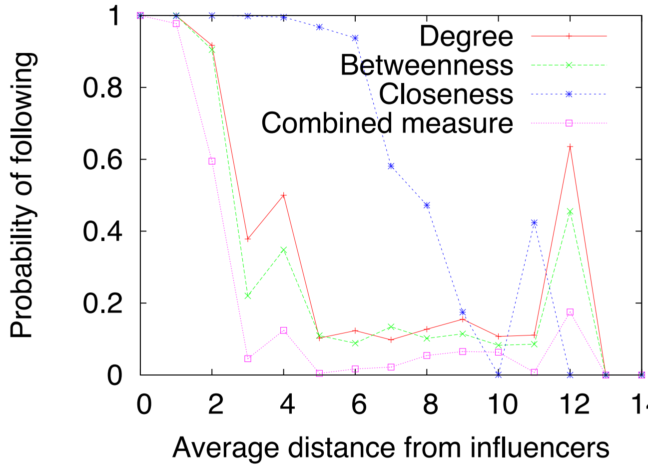

Fig. 4: The variation of the probability of following against the average distance from seeds placed at random positions in a scale-free network. The opposing seeds are placed in the order of (i) degree, (ii) betweenness from the hub, (iii) closeness centrality, (iv) combination of both of 2 & 3. The results are averaged over 20 independent runs.

TABLE III: The average probability of following the randomly placed seeds in different topologies. The opposing seeds are placed according to four different configurations.

Opposing influencer configuration	Average following probability
Degree	0.30
Betweenness (seeds)	0.19
Closeness	0.95
Combined measure (2 & 3)	0.04

proportional to their distance from the influencers. Based on this model, we show that scale-free networks facilitate social influence compared to ER random and lattice networks.

We then extend this model to scenarios where there are multiple and opposing seeds. We propose a method to optimally place the negative seeds to minimise the influence of the original set of positive influencers. In this method, the opponents are placed according to the order of a combined centrality measure, which is the average of the betweenness centrality from the original seeds and the closeness centrality of the nodes within the network. We demonstrate that in general, placing the opponents in the order of betweeness centrality from the original seeds, thereby 'interfering' their influence to the rest of the network, is more effective than choosing the nodes that have higher closeness centrality within the entire network to place the rivalling influencers. Moreover, the proposed combined centrality measure performs best in maximising the negative influence, when the original seed is placed at the hub, or when multiple original seeds are randomly distributed.

To our understanding, this is the first attempt to model the social influence using bounded rationality and the QRE model. The applications of such a model could be myriad, especially as it allows the computation of social influence in a computationally efficient manner. Moreover, as it is based on game theoretic principles, it allows the payoff of following an influencer or adopting a strategy to be a key variable in the modelling, which is not present in the standard social influence models. Countering the influence of an existing network is a critical problem that may have many applications in scientific, social and political networks. Thus, the combined centrality measure of placing opponent nodes and strategies may be quite useful in negatively affecting existing social influence. This model can be even applied to model social influence in scenarios where there are multiple types of influencers and strategies, such as in a political campaign.

As future work, this model may be extended to model the influence in strategic decision making among players in a social network. Also, the possibility of utilizing other centrality measures such as the eigenvector centrality and k-core centrality could be evalutated in placing the opponent seeds. While betweenness centrality and closeness centrality provide the most evident centrality measures to place the seeds, there may be other centrality measures that could be useful for this purpose. While the simulations conducted here are mainly based on theoretical network models such as the ER model and the Scale-free model, applying the given method on real world networks would help to further validate the applicability of the proposed influence model.

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