LOW-FEEDBACK ADAPTIVE-RATE MIMO SPATIAL MULTIPLEXING WITH ZERO-FORCING DETECTION

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ABSTRACT

In this paper, a rate-adaptive spatial multiplexing system with zero-forcing (ZF) detection is proposed for spatially-white MIMO Rayleigh fading channels, feeding back only constellation selection information, which has the advantage of low feedback compared to classical approaches. We evaluate the spectral efficiency achieved by rate adaptation according to the received SNR and we compare the achieved rates with that of the non-adaptive case and with the Shannon capacity. It is shown that the proposed rate-adaptive spatial multiplexing MIMO system achieves a significant gain in spectral efficiency compared to its non-adaptive counterpart in the SNR region of practical interest. An evaluation of the effect of feedback delay on the performance is also undertaken. It is observed that since an increasing diversity order mitigates the effect of fading and makes the channel behave more similarly to an AWGN channel, it also mitigates the effect of delay.

1. INTRODUCTION

Information-theoretic analysis shows that the capacity of multiple-input multiple-output (MIMO) systems with n_T transmit antennas and n_R receive antennas scales linearly with $\min\{n_T, n_R\}$ in Rayleigh spatially white (rich scattering) fading channels even without channel state information at the transmitter (CSIT) [1]. Without CSIT, it is common to apply uniform power allocation among transmit antennas. In practice, when $\min\{n_T, n_R\}$ gets large, MIMO spatial multiplexing systems with uniform power allocation and fixed rate can achieve a large portion of the capacity by simultaneously transmitting independent data streams from different antennas [1].

On the other hand, research results show that with feedback from the receiver, the performance of MIMO systems can be improved [2]. Various schemes have been developed to improve diversity or to boost data rates based on channel mean or covariance feedback, which requires a large amount of side information. Since practical feedback links have limited bandwidth, there is much motivation to use schemes with reduced feedback. In [3] and [4], power- and rate-adaptive single-input single-output (SISO) systems are studied. In [5], rate and power adaptations are applied to MIMO spatial multiplexing systems. In [3], it is pointed out that rate adaptation is the key to achieving high spectral efficiency with reasonable system complexity and it is shown that rate-adaptive schemes using only several constellations can achieve significant gains in data rates over the non-adaptive schemes, by feeding back only constellation selection information. We hereafter refer to this type of feedback as low feedback (as opposed to feedback of channel matrices [2]). Clearly, low feedback leads to performance loss, but schemes with low feedback still achieve significant gains over fixed-rate systems. In [6], rate adaptation based on low feedback is applied to orthogonal space-time block coded systems.

Based on results in [8], we have found that a spatial multiplexing system with zero-forcing (ZF) detection lends itself to convenient bit error rate (BER) performance analysis and low-feedback rate adaptation. So here we propose to apply the same rate adaptation scheme as in [3] and [6] to spatial multiplexing systems with ZF detection. ZF detection performance approaches that of linear minimum mean squared error (LMMSE) detection at high SNR. Following similar analysis as in [3] and [6], we evaluate the spectral efficiency improvement achieved by rate adaptation according to instantaneous received SNR and compare the achieved rates with the Shannon capacity and those of non-adaptive systems. Similar to [6], the effect of delay on system BER performance is also considered. It is observed that since an increasing diversity order mitigates the effect of fading and makes the channel behave more similarly to an AWGN channel, it also mitigates the effect of delay.

The rest of the paper is organized as follows. Section 2 describes the system model. In Section 3, we evaluate the performance of the proposed rate-adaptive spatial multiplexing system with ZF detection. Numerical results are provided in Section 4 and conclusions are drawn in Section 5.

2. SYSTEM MODEL

Consider a $n_R \times n_T$ MIMO spatial multiplexing system with ZF detection. The MIMO channel, denoted by $\mathbf{H} = (h_{ji})_{n_R \times n_T}$, is Rayleigh flat fading, spatially white but temporally correlated in general. Assuming the Jakes model [3], the temporal magnitude correlation of two channel realizations separated τ seconds apart is given by $\rho = J_0^2(2\pi f_d \tau)$, where $J_0(\cdot)$ denotes zero-th order Bessel function of the first kind and f_d is the maximum Doppler frequency. The channel gain h_{ji} from transmit antenna i to receive antenna j is assumed to be circularly symmetric zero-mean complex Gaussian with unit variance: $\mathcal{CN}(0,1)$. Independent and identically distributed (i.i.d) data streams are transmitted from different antennas. The total transmit power P is uniformly distributed among the transmit antennas. The $n_R \times 1$ received signal vector $\mathbf{r}$ is given by

$$\mathbf{r} = \sqrt{P/n_T}\mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is $n_R \times 1$ spatially and temporally white noise vector with each element subject to $\mathcal{CN}(0,\sigma^2)$ and $\mathbf{x}$ denotes the $n_T \times 1$ transmitted signal vector, each element of which is of average unit energy. Perfect synchronization and channel estimation at the receiver are assumed. For ZF detection, we also assume $n_R \geq n_T$. Since the channel is spatially white, $\mathbf{H}$ has full column rank with probability one [7], as required for ZF detection. The instantaneous post-detection signal-to-noise ratio (SNR) denoted by γ can be obtained for each data stream, based on which an appropriate constellation is selected for the corresponding modulator at the transmitter to guarantee a certain target BER. The information about the selected constellation is available at the transmitter via feedback. We assume that the feedback link from the receiver to the transmitter may introduce a delay, but that this link is error-free. The latter can be ensured in practice, e.g., by powerful error-control coding [3]. In the following, both cases with and without delay will be considered.

3. RATE-ADAPTIVE SPATIAL MULTIPLEXING USING ZERO-FORCING DETECTION

Based on (1), the post-detection signal vector is given by [1]

$$\hat{\mathbf{x}} = \mathbf{x} + \mathbf{G}_{ZF}\mathbf{n} \tag{2}$$

where $\mathbf{G}_{ZF} = \sqrt{n_T/P}(\mathbf{H}^\dagger\mathbf{H})^{-1}\mathbf{H}^\dagger$. The average received SNR of the i-th detected symbol in $\hat{\mathbf{x}}$ is

$$\gamma_i = \frac{P/n_T}{\sigma^2[(\mathbf{H}^\dagger\mathbf{H})^{-1}]_{i,i}} \,. \tag{3}$$

Same as in [3], the rate-adaptation scheme will use M-QAM modulations with two-dimensional Gray coding. The constellation size is chosen from the set $\{M_0, M_1, \ldots, M_N\}$, where $M_0 = 0$ stands for no transmission. The received SNR is divided into $N+1$ regions. The region boundaries are given by $\mathbf{a} = [\alpha_0, \alpha_1, \ldots, \alpha_N, \alpha_{N+1}]$ with $\alpha_0 = 0$ and $\alpha_{N+1} = \infty$. If the instantaneous received SNR γ_i for the i-th data stream falls between α_n and α_{n+1}, the constellation of size M_n is selected. To inform the transmitter of the selection, we need to feedback $\log_2(N+1)$ bits per data stream (or per transmit antenna) for each adaptation, or $n_T \log_2(N+1)$ bits in total.

Since the data streams from different antennas are i.i.d., we can drop the index i. Thus without loss of generality, we consider only data stream 1 in the subsequent analysis. Let γ_s denote the average SNR per transmit antenna, i.e., $\gamma_s = P/(\sigma^2 n_T)$, so that from (3), we have $\gamma = \gamma_s/[\mathbf{H}^\dagger\mathbf{H})^{-1}]_{1,1}$. It can be shown [8] that γ has a Gamma distribution

$$p_\gamma(\gamma) = \frac{\gamma^{m-1}}{\gamma_s^m \Gamma(m)} \exp(-\gamma/\gamma_s) U(\gamma) \tag{4}$$

where $m = n_R - n_T + 1$ is the diversity order, $\Gamma(n) = (n-1)!$ and $U(\cdot)$ is the unit step function.

For $M \geq 4$ and $BER \leq 10^{-2}$, the BER of coherent M-QAM with Gray coding over an additive white Gaussian noise channel (AWGN) can be approximated by [3]

$$BER(M,\gamma) \simeq 0.2 \exp\left(-\frac{3\gamma}{2(M-1)}\right). \tag{5}$$

The above formula provides an upper bound for $M \geq 4$ and a lower bound for $M = 2$ (BPSK). The exact BER for BPSK is $Q(\sqrt{2\gamma})$, where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-u^2/2}du$. Suppose we are given a target average BER, BER_0. Then the region boundaries can be chosen as [3]

$\alpha_1 = 0.5[Q^{-1}(BER_0)]^2$ for BPSK (if used),

$\alpha_n = \frac{2}{3}K_0(M_n - 1)$ for M_n-QAM ($M_n \geq 4$),

where $K_0 = -\ln(5BER_0)$ and $Q^{-1}(\cdot)$ is the inverse Q-function.

In the following study, we first consider the case with zero-delay feedback. Based on the above rate adaptation, the achievable spectral efficiency (in bits/sec/Hz/transmit antenna or simply bps/Hz/antenna) can be calculated as follows [6]

$$\overline{R} = \sum_{n=1}^{N} \frac{R_n}{\Gamma(m)} \left\{\Gamma(m;\alpha_n/\gamma_s) - \Gamma(m;\alpha_{n+1}/\gamma_s)\right\} \tag{6}$$

where $R_n = \log_2(M_n)$ and $\Gamma(x;t) = \int_t^\infty e^{-y}y^{x-1}dy$. The average BER can be obtained using [6]

$$\overline{BER} \simeq 0.2\overline{R}^{-1} \sum_{n=1}^{N} \frac{R_n \beta_n^m}{\Gamma(m)} \{\Gamma(m;\frac{\alpha_n}{\beta_n\gamma_s}) - \Gamma(m;\frac{\alpha_{n+1}}{\beta_n\gamma_s})\} \tag{7}$$

where $\beta_n = \frac{2(M_n-1)}{3\gamma_s + 2(M_n-1)}$.

Second, we consider the case with feedback delay τ. In this case the signal constellation is selected at time t, but the demodulation is performed at time $t+\tau$. Based on the channel realization at time $t+\tau$, the previously selected constellations may not satisfy the target BER_0. Therefore, the performance will be degraded depending on the delay. We assume stationary channel so that all the realizations have the same distribution. The received SNRs at time t and $t+\tau$ are denoted as γ and γ_τ, respectively. Since $\overline{R}$ is calculated according to (6) and γ, γ_τ have the same distribution, the spectral efficiency is not affected by feedback delay. Thus we will focus on BER degradation. The average BER in this case can be formulated as [6]

$$\overline{BER}' \simeq 0.2\overline{R}^{-1}\sum_{n=1}^{N}\frac{R_n\,\beta_n^m}{\Gamma(m)}\{\Gamma(m;\frac{\alpha_n}{b_n\gamma_s})-\Gamma(m;\frac{\alpha_{n+1}}{b_n\gamma_s})\} \tag{8}$$

where $b_n=\frac{2(M_n-1)+3\gamma_s(1-\rho)}{2(M_n-1)+3\gamma_s}$ and ρ is given in Section 2.

It is worth noting that under unfavorable channel conditions, M_0 is selected and outage occurs. So in practice we need to buffer the data [3].

4. NUMERICAL RESULTS

We will use the set of constellation sizes $\{0,2,4,16,64\}$, the Doppler frequency f_d = 100 Hz and $BER_0=10^{-3}$. Note that for fair comparison, we should keep the total transmit power the same for cases with different numbers of transmit antennas. Here we keep n_T = 2 and let $n_R\in\{2,4\}$.

A. *Zero-delay feedback*:

Figs. 1 and 2 show both the analytical and simulation results for n_R = 4 and 2, respectively. Fig. 1 shows that the average BER, $\overline{BER}$, is strictly below 10^{-3} since in our design we maintain the instantaneous BER below the target. The simulation results agree well with the analytical results. From Fig. 2, we can see that at γ_s = 20 dB, our scheme achieves a spectral efficiency of 5.25 bps/Hz/antenna and 3.27 bps/Hz/antenna for $n_R=4$ and $n_R=2$ ($n_T=2$), respectively, whereas its non-adaptive counterpart with $n_R=4$ and $n_R=2$ achieves only 2 bps/Hz/antenna. The achievable spectral efficiency can be improved by enlarging the constellation size set, at the expense of complexity. The curve for the non-adaptive 4QAM-modulated system in Fig. 2 is obtained as follows. First, using (4) and (5), the average BER, $\overline{BER}_{na}$, is

$$\overline{BER}_{na}\simeq\frac{0.2}{(1+\gamma_s/2)^m} \tag{9}$$

from which we can find the minimum γ_s which satisfies $\overline{BER}_{na}\leq BER_0$. For $BER_0=10^{-3}$, this value of γ_s is about 10 dB. We then plot the curve beginning with $\gamma_s=10$ dB. Now we compare the achieved spectral efficiency with the Shannon capacity (bps/Hz/antenna) for spatial multiplexing with uniform power allocation, which is [1]

$$C=\frac{1}{n_T}E\left\{\log_2\det\left[\mathbf{I}_{n_T}+\gamma_s\cdot(\mathbf{H}^\dagger\mathbf{H})\right]\right\} \tag{10}$$

where $\mathbf{I}_n$ denotes the $n\times n$ identity matrix. The capacity formula assumes infinitesimal granule in the constellation size (continuous rate). Fig. 2 shows that for a spectral efficiency of 4 bps/Hz/antenna, the proposed system with $n_T=2$ and $n_R=4$ is 9.4 dB away from the Shannon capacity, and with $n_T=2$ and $n_R=2$, it is about 11.1 dB away from the capacity. Note that since we use finite constellation sizes (discrete rate) at the transmitter, the channel input is not Gaussian. This incurs an inherent rate loss for the proposed scheme.

B. *Nonzero-delay feedback*:

Figs. 3 and 4 show the average BER performance with feedback delay. As in [6], we normalize the delay τ to the Doppler frequency f_d. We set $f_d\cdot\tau=0.05$ in Fig. 3. By comparing Fig. 3 with Fig. 1, we can see the degradation of average BER performance due to delay. In Fig. 4, we set $\gamma_s=17$ dB. Thus even with $f_d\cdot\tau$ equal to 0.03 and with $n_T=n_R=2$ (i.e., $m=1$, no diversity), the adaptive system still satisfies the target BER. Also, Figs. 3 and Fig. 4 show that higher diversity order ($m=n_R-n_T+1$) leads to lower sensitivity to delay. Similar phenomena have been observed in [3] and [6]. This is expected, since an increasing diversity order mitigates the effect of fading, making the channel behave more similarly to an AWGN channel, and thus mitigates the effect of delay.

5. CONCLUSIONS

In this paper, a rate-adaptive spatial multiplexing MIMO system with zero-forcing detection is proposed and studied. It is shown that for zero-delay feedback and with target $BER_0=10^{-3}$, at average SNR per transmit antenna γ_s = 20 dB, the adaptive scheme achieves a spectral efficiency of 5.25 bps/Hz/antenna and 3.27 bps/Hz/antenna for n_R = 4 and n_R = 2 (n_T is set to 2), respectively, whereas its non-adaptive version using 4-QAM with n_R = 4 and n_T = 2 achieves only 2 bps/Hz/antenna. Since an increasing diversity order mitigates the effect of fading and makes the channel behave more similarly to an AWGN channel, it also mitigates the effect of delay. It is demonstrated by simulation results that even without diversity, the adaptive system can still tolerate a $f_d\cdot\tau$ as large as 0.03 when γ_s = 17 dB and $BER_0=10^{-3}$.

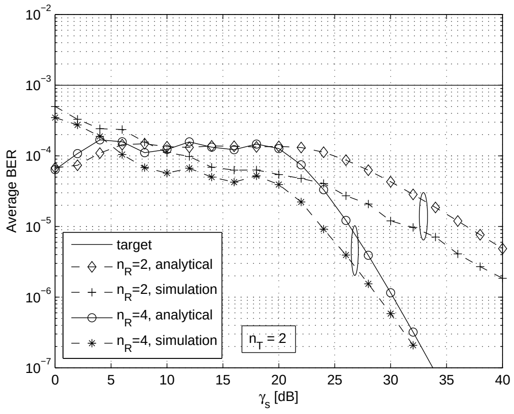

Fig. 1. Average BER, zero-delay feedback

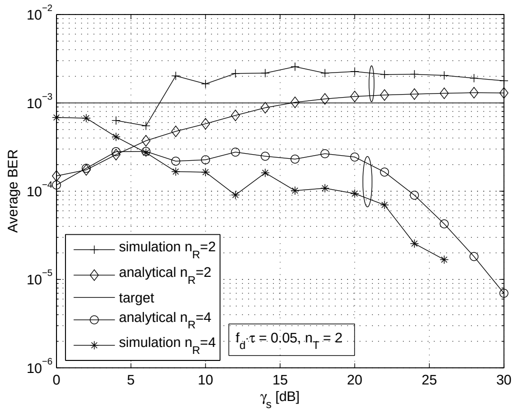

Fig. 3. Average BER, delayed feedback

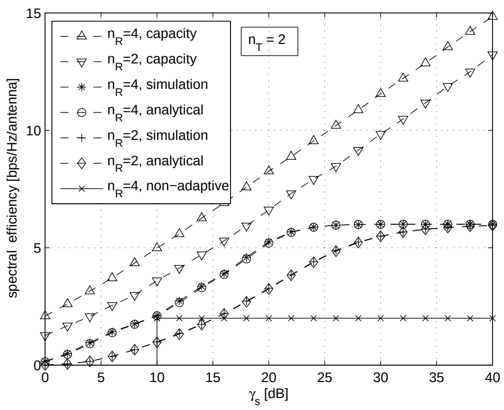

Fig. 2. Spectral efficiency

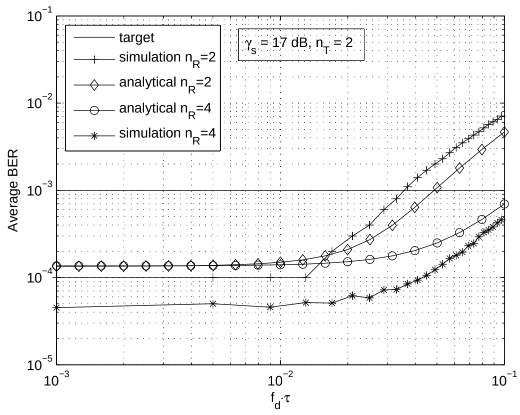

Fig. 4. Performance degradation with delay

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