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Node assortativity in complex networks: An alternative approach

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Abstract

Assortativity quantifies the tendency of nodes being connected to similar nodes in a complex network. Degree Assortativity can be quantified as a Pearson correlation. However, it is insufficient to explain assortative or disassortative tendencies of individual nodes or links, which may be contrary to the overall tendency of the network. A number of 'local' assortativity measures have been proposed to address this. In this paper we define and analyse an alternative formulation for node assortativity, primarily for undirected networks. The alternative approach is justified by some inherent shortcomings of existing local measures of assortativity. Using this approach, we show that most real world scale-free networks have disassortative hubs, though we can synthesise model networks which have assortative hubs. Highlighting the relationship between assortativity of the hubs and network robustness, we show that real world networks do display assortative hubs in some instances, particularly when high robustness to targeted attacks is a necessity.

Keywords:

1 Introduction

The study of complex networks is a dominant trend in recent research that transcends domain boundaries. Assortativity is a much studied concept in the topological analysis of complex networks [11, 13]. Assortativity has been defined to quantify the tendency in networks where individual nodes connect with other nodes which are similar to themselves [11]. Thus, a social network of people tends to be assortative, since people often prefer to be friends with, or have links to, other people who are like them. A food web could be argued as disassortative, because predator and prey are unlikely to be similar in many respects. However, it is clear that the assortativity of a network needs to be defined in terms of a particular attribute of nodes in that network. A social network could be assortative when the considered attribute is the age of people, because people tend to be friends with other people similar to their age: however, the

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same network could be disassortative, when the gender of individuals is the considered attribute. Degree assortativity is the most common form of assortativity used in network analysis, whereby similarity between nodes is defined in terms of the number of connections the nodes have. Degree assortativity can be defined and quantified as a Pearson correlation [11, 8]. It has been shown that many technological and biological networks are slightly disassortative, while most social networks, predictably, tend to be assortative in terms of degrees [11, 17]. Recent work defined degree assortativity for directed networks in terms of in-degrees and out-degrees, and showed that an ensemble of definitions are possible in this case [8].

It is apparent from the above discussion that assortativity is a network level quantity. However, individual nodes can also exhibit assortative or disassortative tendencies. For example, in some networks, those nodes which are highly connected tend to be connected to other nodes which are also highly connected. This is sometimes called the rich-club phenomena [19, 20]. This does not automatically imply the network is assortative because the vast majority of the nodes, which are not the richest in terms of degrees, can be either assortatively or disassortatively connected. Indeed, it has been shown that the Internet Autonomous System (AS) networks, which are disassortative at the network level, show rich club phenomena [19, 20, 5]. However, understanding the assortative tendencies of individual nodes in networks is important, to classify networks, to characterise nodes, and to understand the functional importance of nodes, among other reasons [13, 15].

A number of recent studies have attempted to classify the assortativity of individual nodes, and indeed individual links, in networks. Piraveenan et al introduced the concept of 'local assortativity' for this purpose [13, 15] and have proposed a number of definitions. However, all these definitions make a distinction between assortative and disassortative nodes (or links) based on the 'expected degree' (or expected remaining degree) of the network. We will argue in this paper that the use of this quantity as a pivot may not be always ideal. In this paper, we propose an alternative approach, based on the simple assumption that all nodes in a network are, to various levels, disassortative, and the average degree difference of a node and its neighbours can, therefore, be used as a measure of (dis) assortativity. We will argue that this approach is more intuitive and computationally less expensive than the other current definitions.

The rest of the paper is organised as follows: In the following section, we will review the concept of assortativity and some of the existing measures to quantify node (and link) assortativity. Then we will introduce our alternative approach to measure node assortativity, and apply it to some canonical networks. In the next section, we will demonstrate the utility of the measure by applying it to a number of simulated and real world networks. Then we will discuss the implications of our results and draw observations and conclusions. In the final section we will summarise the paper.

2 Background

Degree assortativity has been defined by Newman, as a Pearson correlation between the 'expected degree' distribution q_k, and the 'joint degree' distribution $e_{j,k}$ [11]. The expected degree distribution is the probability distribution of traversing the links of the network, and finding nodes with degree k at the end of the links. Similarly, the 'joint degree' distribution is the probability distribution of a link having degree j on one end and degree k on the other end. In the undirected case, the normalized Pearson coefficient of $e_{j,k}$ and q_k gives us the assortativity coefficient of the network, r.

Assortativity coefficient can be defined as [12, 11]:

$$r = \frac{1}{\sigma_q^2}\left[(\sum_{jk} jk e_{j,k}) - \mu_q^2 \right] \tag{1}$$

where $e_{j,k}$ is the joint probability distribution of the excess degrees of the two nodes at either end of a randomly chosen link. Here μ_q and σ_q are the expected value or mean, and standard deviation, of the excess degree distribution q_k. If a network has perfect assortativity ($r = 1$), then all nodes connect only with nodes with the same degree. If the network has no assortativity ($r = 0$), then any node can randomly connect to any other node. If a network is perfectly disassortative ($r = -1$), all nodes will have to connect to nodes with different degrees. A star network is an example of a perfectly disassortative network, and complex networks with star 'motifs' in them tend to be disassortative.

Now let us review the efforts to quantify assortativity at node level. Piraveenan et al [13, 15] proposed the quantity of 'local assortativity' for this purpose. The local assortativity of node v in an undirected network is defined as:

$$\rho_v = \frac{\alpha_v - \beta_v}{\sigma_q^2} = \frac{j\,(j+1)\left(\overline{k} - \mu_q\right)}{2M\sigma_q^2} \tag{2}$$

where j is the node's remaining degree, $\overline{k}$ is the average remaining degree of its neighbours, and $\sigma_q \neq 0$. Note that, in this definition, the sign of the local assortativity (positive or negative) is determined by the difference between the average excess degree ($\overline{k}$) of the neighbours and the global average excess degree (μ_q). If the neighbours' average is higher, then the node is assortative. If the global average is higher, the node is disassortative. Therefore, the local assortativity can also be interpreted as a scaled difference between the average excess degree of the node's neighbours and the global average excess degree. In other words, a node is locally assortative if it is surrounded by nodes with 'comparatively' high degrees.

Note that in the definition above, the quantity μ_q plays a pivotal role in determining whether a node is assortative or disassortative. If the average neighbour degree is higher than the 'expected' node degree, the node is simply considered assortative, and vice versa. Therefore, a peripheral node which is connected to identical peripheral nodes will be disassortative, while a hub node which is part of a rich club is considered assortative, even if the members of the rich club have large scale variation in terms of degrees. It can be argued that this is somewhat counter-intuitive: even though somewhat meaningful in describing the 'local neighbourhood' of a node, it goes against the global definition of assortativity.

Thus, an edge which connects two nodes with degrees higher than μ_q will be considered assortative, even if those degrees are themselves very different, and an edge which connects a node with degree higher than μ_q with another node with degree lower than μ_q will be considered disassortative, even if those degrees are themselves very close. This is clearly counter-intuitive. Therefore, a definition which does not pivot on the quantity μ_q needs to be proposed. We arrive at such a definition by going back to the fundamental tenet of assortativity, which is the amount of similarity between node attributes.

3 Introduction to the alternative approach

First of all, let us recognise that unlike network assortativity, node assortativity is a relative concept. We cannot decide whether a node is assortative or disassortative by looking at just the node's locality (except in the rare case of a node having all neighbours with exactly the same degree as itself). We need to compare that node with other nodes in the network. It can be intuitively argued however, that node assortativity, in terms of degrees, must mean that a node must be relatively assortative if it has more connections with other nodes with similar degrees, where as a node must be relatively disassortative if it has more connections with other nodes which have dissimilar degrees.

Consider a regular lattice, where all nodes connect only with six other nodes with same degrees. We can say, therefore, that all nodes are assortative. If, such a network is slightly modified such that a node connects with five similar nodes (in terms of degrees) and one dissimilar node, is it assortative or disassortative? It could be argued that it is relatively disassortative. However, a node with similar mixing pattern could be considered relatively assortative in another network topology, where all other nodes are even more disassortative. Therefore, node assortativity is a relative concept. Furthermore, we are in most situations interested in finding out about the relative assortativity of a node only, within its network. Whether it is assortative or disassortative from an 'outside-the-network perspective' is not always meaningful or relevant.

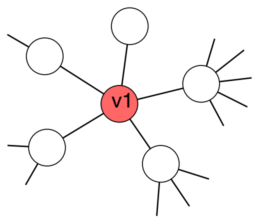


Figure 1: Average neighbour difference, in terms of node degrees. The highlighted node has an average difference of $\delta_v = 2.2$ between its own degree and neighbour degrees.

We can therefore reasonably argue that the average number of 'differences' in node degrees between a node and its neighbour is a fair indicator of the 'disassortativity' of a node. Let us denote this quantity as δ_v. Thus, all nodes in a regular lattice would have $\delta_v = 0$. We concentrate on undirected networks for the reminder of this paper. Consider Fig 1, where node v_1 with degree 5 is connected to five neighbours, which themselves have degrees $6, 4, 3, 2, 1$. Therefore δ_{v1} is $(|6-5|+|4-5|+|3-5|+|2-5|+|1-5|)/5 = 2.2$. We can call this quantity as the 'average neighbour difference', which is a direct indicator of a node's disassortativity. It is given by:

$$\delta_v = \frac{1}{d_v}\sum_{i=1}^{d_v}|d_i - d_v| \tag{3}$$

where d_i is the degree of the node i.

However, this measure would indicate that all nodes in a network are disassortative to various extents, and at best, non-assortative. However, there are many real world networks which are assortative ($r > 0$), though not perfectly so [17]. It would make more sense if the above measure is scaled in such a way that many nodes in such a network can be labelled assortative. We achieve this in the following manner.

First of all, we scale the average neighbour difference values for each node, by dividing it by the sum of such values, S. The scaled values, $\overline{\delta}_v$, will therefore have a sum of $S' = 1.0$[1]. Now we add a scaling factor λ so that some of the nodes become assortative. This scaling factor can be randomly chosen, and in fact acts as a threshold which determines the number of assortative nodes in the network. However, it will be more meaningful if the node assortativity values obtained can be summed to match the network assortativity, r [12]. Therefore, we will choose the scaling factor λ such that $N\,\lambda$ - $S' = r$, where N is the number of nodes. This will result in

$$\lambda = \frac{r + S'}{N} \tag{4}$$

That is

$$\lambda = \frac{r + 1}{N} \tag{5}$$

Then the node assortativity of node v can be calculated as $\breve{\delta_v} = \lambda - \overline{\delta}_v$. For a perfectly assortative network, $\lambda = \frac{2}{N}$, while for a perfectly disassortative network, $\lambda = 0$, thus making $\breve{\delta_v} = -\overline{\delta}_v$. $\overline{\delta}_v$, by definition, is always positive or zero. Thus a perfectly disassortative network will have no assortative nodes, which is intuitively right. All other networks will have some assortative nodes with $\breve{\delta_v} \geq 0$.

4 Application to Some canonical networks

Now let us apply this definition of node assortativity in some trivial cases. Let us look at two synthesized networks from most commonly used network models, namely an Erdos-Renyi random network and a scale-free network generated by preferential attachment [2]. In both cases, we find that there are peripheral nodes which can be assortative or disassortative, however the hubs are largely disassortative. It appears however that in ER networks, the middle level hubs are even more assortative than the peripheral nodes. This is probably due to the fact that there is no huge variation in degrees among nodes in random networks, whereas in scale-free networks where the variation of degrees is more pronounced, assortativity seems to largely decrease with node degree.

Let us turn aside briefly to address another important consideration. Rich club phenomena [20, 16, 5] is a well known occurrence in many complex networks (such as Internet AS level networks), where the largest hubs have the tendency to connect to each other. However, it is vital to understand if such hubs have the most links among themselves, or most of their links are actually to peripheral nodes, while maintaining a higher than average number of links among themselves. Node assortativity can be used as a tool to identify this.

We use the following example to demonstrate that our definition of node assortativity can indeed result in assortative hubs. It can be observed by inspection that the network shown in Fig 4 (a) has a strong 'rich club'. The corresponding node assortativity values are shown in Fig 4 (b), and it could be seen that the hubs are indeed assortative (Note that due to symmetry in the design, there are several overlaps of points in the distribution plot). Thus the assortativity of hubs in this network, and the corresponding strong rich club, is captured by our definition. We will revisit the topic of rich-clubs in real world networks in later sections.

[1]The exception to this scaling is when all δ_v are zero, and node assortativity would have to be defined via a special case. This can only happen in a perfect regular lattice.

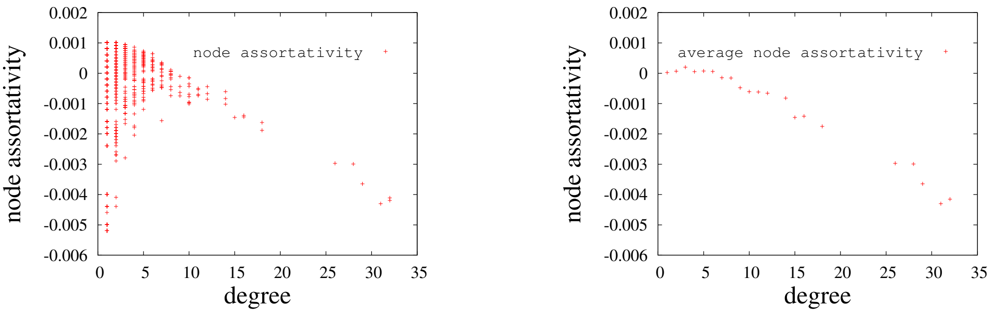


Figure 2: Node assortativity distribution of a Preferential Attachment-based synthetic scale free network of size $N = 1000$ nodes and $M = 3000$ links.

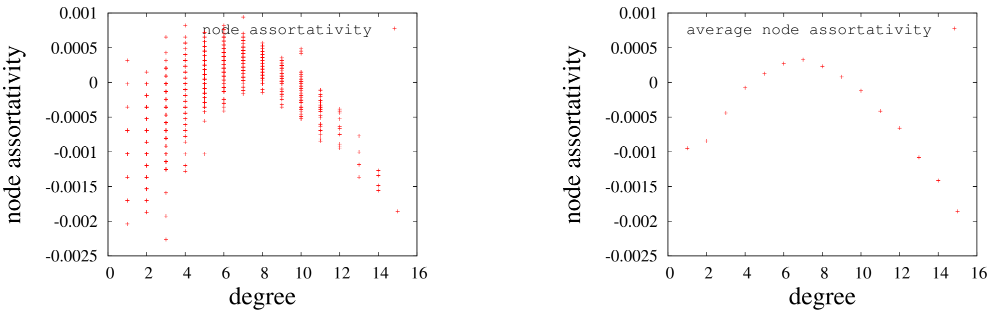


Figure 3: Node assortativity distribution of an Erdos-Renyi random network of size $N = 1000$ nodes and $M = 3000$ links

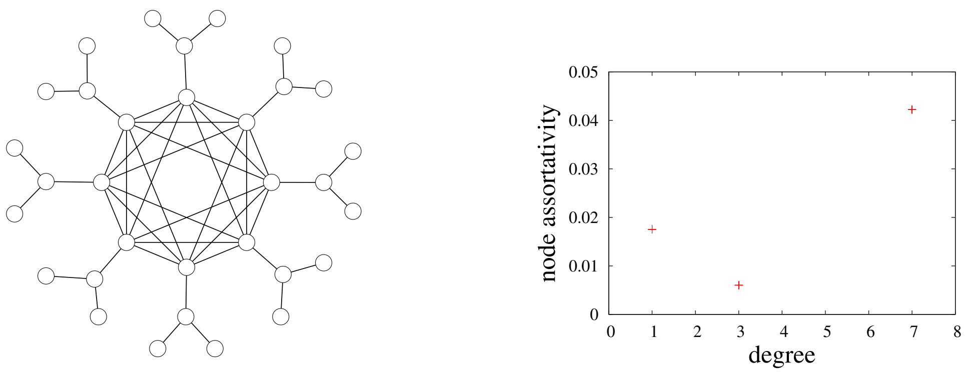


Figure 4: (a). A model network with assortative hubs, (b). A model network with assortative hubs - node assortativity distribution

5 Node assortativity of real world networks

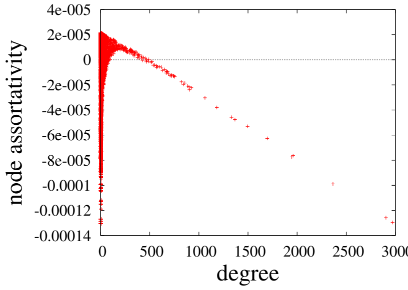
Now let us turn our attention to some real-world networks. We will consider networks from biological, technical, and social domains. Our main aim here is to see if there is any correlation between node degree and node assortativity, and if there is, do the patterns vary from one network group to another. For example, we would like to find out, in these real world networks (i) Are the hubs assortative, disassortative, or both? (ii) Do networks which display the so called rich-club phenomena demonstrate assortative hubs? (iii) Are the majority of the peripheral nodes assortative or disassortative? (iv) Are there any correlations between overall network assortativity and the assortativity of the hubs? etc.

We therefore generated distributions with node degree on one axis and node assortativity on the other axis. Among technical networks, we considered Internet Autonomous System level networks [14], where nodes represent an Autonomous System present in the Internet and the links represent a commercial agreement between two Internet Service Providers (who own the two ASs)[14]. Among biological networks, we considered Gene Regulatory Networks, where the nodes are genes, and the links are the inhibitory or inducing effects of one gene on the expression of another gene [7, 4]. Among social networks, we considered collaboration networks, where the nodes are researchers and links denote collaboration between these researchers. [10]. We also considered Foodwebs which are Ecological networks [1]. Finally, as a poignant one-off example, we also considered a human cortical network. The term *cortical networks* is not a standard term in complex network literature, like the terms used to denote other types of networks in this section. By this term we denote the network of dependencies between various regions of the cerebral cortex (in a set of primates)[18, 9]. The nodes are regions in the cortex, and the links are functional dependencies. Note that the nodes are *not* individual neurons. Some of our results are shown in figures 5,6.

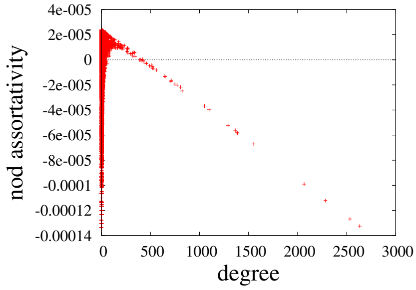
Quite surprisingly, we see from the figures that all of the real world networks show disassortative hubs, regardless of their domain. The peripheral nodes are both assortative and disassortative. This is true even for Internet AS networks which demonstrated the rich-club phenomena, and the Human cortical network which has a high node-to-link ratio (27.2 links per node). However, this cannot be a feature of the definition since we had seen some model networks where the hubs are strongly assortative.

We mentioned earlier that we consider node assortativity a relative quantity within the context of the network. Therefore, we must interpret the results as, all hubs are being disassortative compared to many of the peripheral networks. This is true regardless of the value of λ chosen, since it is comparative. It would appear that our results are in contrast with the results obtained by rich club measures on similar networks [20, 16]. In particular, it has been observed that the Internet AS networks show the 'Rich-Club phenomena', where there is a higher density of connections among hubs. This might appear to imply that the hubs need to be assortative. Therefore, it could be asked, do the hubs seem disassortative only because they and their neighbours have high degrees and the average degree differences are, therefore, amplified? We set out to reconcile this seeming contrast next.

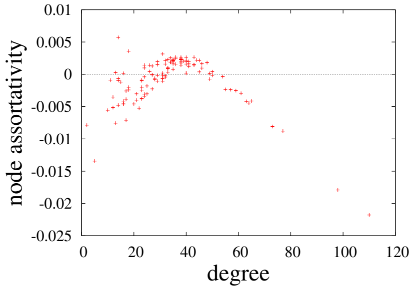
The rich-club connectivity is defined as an average connectivity of nodes that have more than a specified number of degrees [20, 5]. In particular, a ranking system is used on nodes based on degrees, and the connectivity of subgraphs with nodes having a certain percentile rank or above is measured. Therefore, we modify the assortativity and node assortativity definition based on *ranks* of the nodes as well, which are computed based on node degrees, rather than node degrees themselves. Therefore we calculate network assortativity as



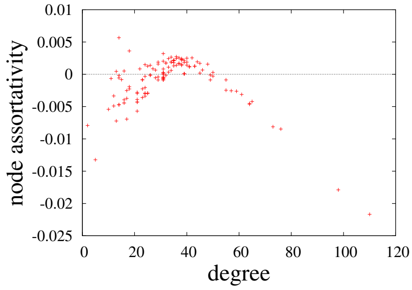
(a) Node assortativity distribution of Internet AS level network - 2011. (r: -0.212)



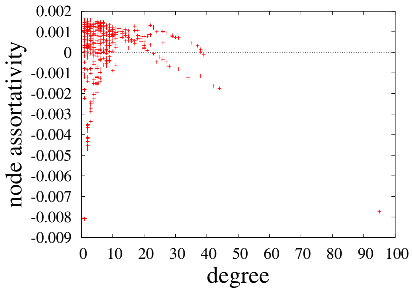
(b) Node assortativity distribution of Internet AS level network - 2010. (r: -0.207)



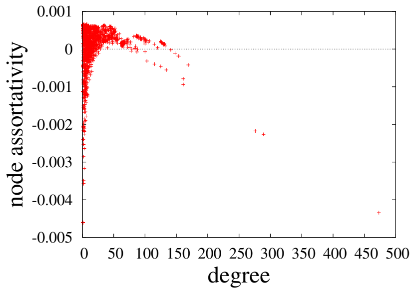
(c) Node assortativity distribution of Bay dry food web. (r: -0.115)



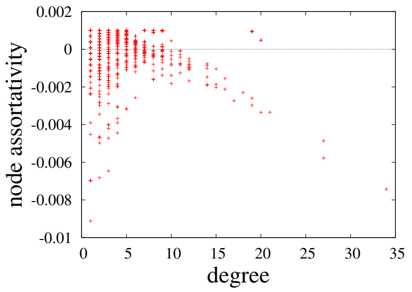
(d) Node assortativity distribution of Bay wet food web. (r:-0.122)



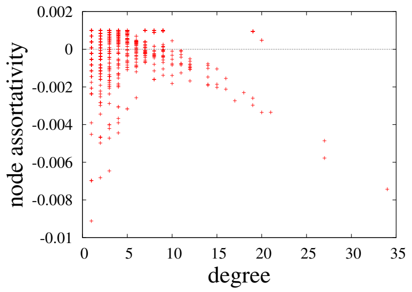
(e) Node assortativity distribution of C. elegans Gene Regulatory Network. (r: -0.087)



(f) Node assortativity distribution of Human Gene Regulatory Network. (r: -0.033)



(g) Node assortativity distribution of hep-theory collaboration Network. (r: 0.294)



(h) Node assortativity distribution of network science collaboration Network. (r: 0.462)

Figure 5: Node Assortativity vs Node Degrees plot for some real world networks.

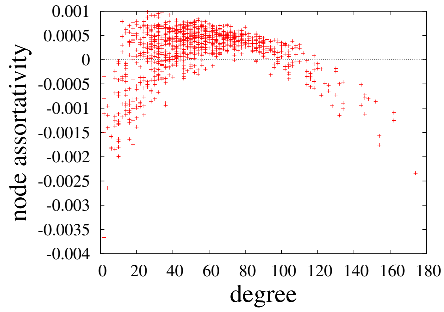


Figure 6: Node assortativity distribution of Human Cortex Network. (r: 0.173)

$$r^{rank} = \frac{M^{-1}}{\sigma_q^2}\left[\sum_{e\in E}(j_e^{rank} - \mu_q^{rank})(k_e^{rank} - \mu_q^{rank})\right] \tag{6}$$

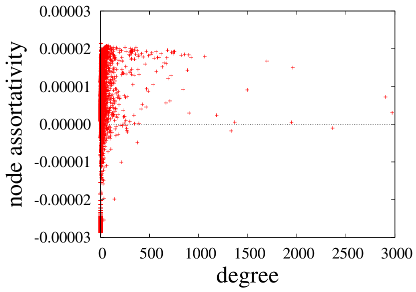
where M is the number of links and E is the set of links. j_e^{rank}, and k_e^{rank} denote respectively the *reverse ranks* of nodes at either end of link e. We used reverse ranks so that the largest hub will have the highest value. The mean and standard deviation μ_q and σ_q are also calculated in terms of the reverse-rank, rather than degrees themselves. Now the average neighbour difference, in terms of nodes, can be calculated for each node as

$$\delta_v^{rank} = \frac{1}{d_v}\sum_{j=1}^{d_v}|rank_j - rank_v| \tag{7}$$

and node assortativity can be calculated as described above, using the rank-based assortativity r^{rank} to calculate the scalar λ.

Using this rank-based definition, we analysed the exact same set of real world networks which were described in section 5. Our results are shown in figures 7,8. We found that there is not much qualitative difference in our results. The foodwebs though, notably show a different pattern now, where the provincial hubs are the most assortative. Regardless, most real world networks still show disassortative hubs, which must now clearly mean that most of the connections of these hubs must be for relatively peripheral nodes. We can rarely see however, that some hubs can be slightly assortative, as in the case of the hep-theory collaboration network, and Internet AS networks. Even here, many peripheral nodes are much more assortative than these hubs. In any case, as mentioned before, we are more interested in comparing hubs to peripheral nodes, than looking at the absolute assortativity values of the hubs, since those can shift subject to the choice of λ. We do not observe any network where there is a tendency for the hubs to be more assortative than peripheral nodes (i.e the assortativity values increasing with degrees). The overall tendency is still that of assortativity values reducing with node degree.

In the case of Internet therefore, we can surmise that, while the hubs may have a higher link density compared to all nodes take together, many links from individual hubs are indeed to peripheral nodes. Therefore the hubs are not highly 'assortative' in the true sense. It appears there are not too many real world networks with assortative hubs. We however see an arguably qualitative difference in the Human Cortical networks, where there is some tendency for the hubs to be as assortative as the most assortative peripheral nodes. That is, the assortativity



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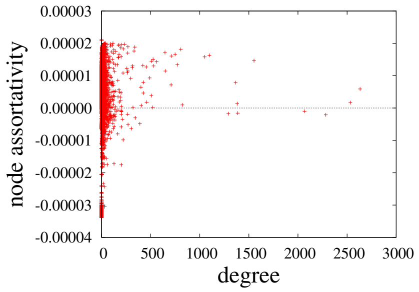
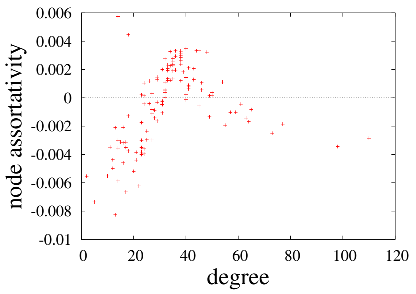
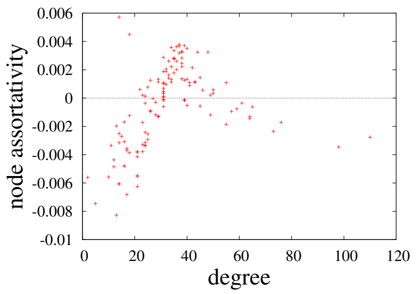


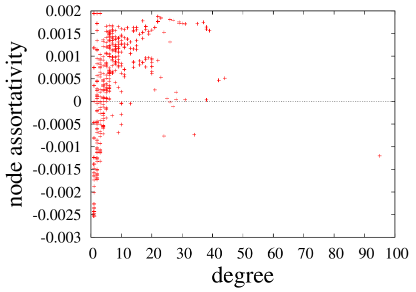
(b) Node assortativity distribution of Internet AS level network - 2010. (r: -0.294)



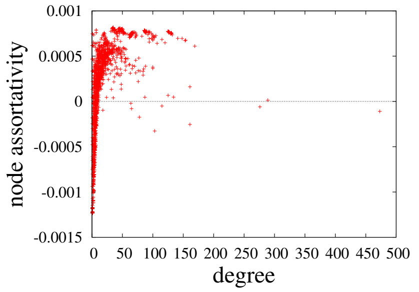
(c) Node assortativity distribution of Bay dry food web. (r: -0.064)



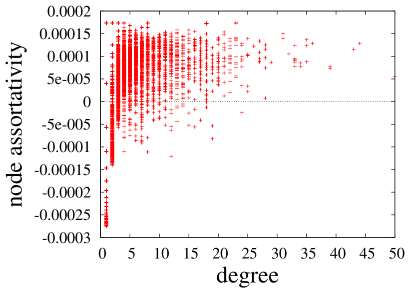
(d) Node assortativity distribution of Bay wet food web. (r: -0.06)



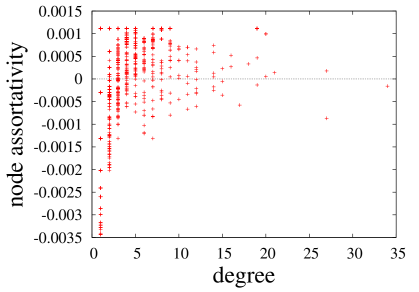
(e) Node assortativity distribution of C. elegans Gene Regulatory Network. (r: 0.129)



(f) Node assortativity distribution of Human Gene Regulatory Network.(r: 0.245)



(g) Node assortativity distribution of hep-theory collaboration Network. (r: 0.319)



(h) Node assortativity distribution of network science collaboration Network. (r: 0.627)

Figure 7: Node Assortativity vs Node Degrees plots using rank-based assortativity

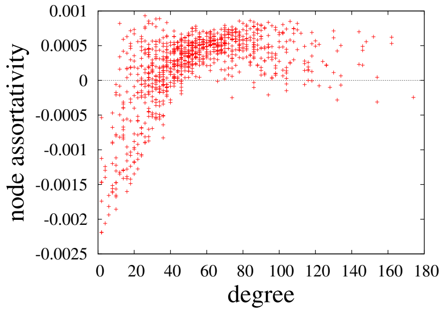


Figure 8: Node assortativity distribution of Human Cortex network based on node rank. (r: 0.206)

does not reduce with node degrees. This is possibly related to the high node-to-link ratio this network displays, and may have implications for the networks' robustness, as we will discuss below.

6 Discussion

It is quite significant that we found most real world networks have (to various extents) disassortative hubs. Yet we also saw that there can be networks with assortative hubs. From these observations arise some important questions: (i) Is it simply a feature of large-scale networks that the hubs need to be disassortative, to maintain connectivity? (ii) Is it a feature of scale-free networks? (iii) Is it a feature of evolved networks, connected with the robustness of networks? (iv) What design or environmental constrains are necessary to make real world networks have (strongly) assortative hubs?

In answering these questions, let us first note that we have by no means done an exhaustive analysis of real world networks. This is a subject of future research and we may indeed find large real world networks with strongly assortative hubs. In answer to question (i), it could be speculated that there can be strongly assortative hubs, if a majority of all links are between hubs. However, this could be harder to achieve, even impossible, with a power law degree distribution. Therefore, it may be that all scale-free networks will show disassortative or non-assortative hubs. More research needs to be done to determine whether a network with a power law degree distribution can be constructed while maintaining assortative hubs.

However, it could be argued that having assortative hubs is good in terms of robustness and attack-tolerance of networks particularly in terms of targeted attacks. It is well known that scale-free networks are comparatively vulnerable to targeted attacks [3, 6]. This is because there are a few large hubs which can be targeted and picked-off. However, if hubs are assortative, then it will increase the robustness of networks because hubs can function as back-ups to each other. Therefore, it will be interesting to understand why assortative hubs have not evolved in many networks which need to be robust. On the other hand, it can be argued that link redundancy is strongly connected to assortative hubs. We have seen that the human cortex network, which displays 994 nodes and 27040 links, has some assortative hubs. The high node-to-link ratio means that there is redundancy in terms of connections, and there is a true 'rich-club' of hubs, backed up by each other. Needless to say, the cortical network is critical for human body and need to be resilient to any type of failures. Sections of brain may be susceptible to illness or failure and it is vital that the failure of 'hubs' in the cortex do not adversely impact the function of the entire cortex. This is one instant where assortative hubs seem to have evolved to achieve robustness to attacks. We may note however that the network is not scale-free, where a scale free exponent (0.491) can be fitted only with a small correlation (of 0.096). The question therefore arises whether attack tolerance is compromised in many real world networks in order to achieve scale-free feature.

7 Conclusions and future work

Assortativity is the tendency among nodes whereby nodes make links with similar nodes. It is useful to quantify the assortativity of individual nodes, to see the correlation between node degree and assortativity, to classify networks, and to understand node functionality. A number of attempts have been made to quantify the 'local' assortativity, however these all pivot on the 'expected remaining degree' (average remaining degree) of the network. We showed that this dependency has its disadvantages. We therefore proposed a set of simple alternative definitions (primarily for undirected networks but easily extendable to directed networks), based on the premise that node assortativity is relative, and the relative disassortativity of nodes can be measured by average neighbour difference of degree or degree-based rank. We first validated our derivations against canonical and synthesized networks before applying them to a set of real world networks.

We observe that, all real world networks show disassortative hubs, when node assortativity is derived from average neighbour difference of degrees. Yet, we had seen model networks which can demonstrate the existence of assortative hubs, given a sufficient relative density of links among them. We therefore surmised that the rich club phenomena observed in some of these networks must mean that while they have a higher than average link density among hubs, the hubs do not necessarily have a majority of connections among themselves (we call these 'weak rich-clubs). To further verify this, we used a node assortativity definition which is based on ranks of nodes. We found that most real world networks still did not have assortative hubs, though some have non-assortative (rather than dis-assortative) hubs. Importantly, even those networks which had non-disassortative hubs did not show the tendency of node assortativity increasing with node degree. The human cortex network was a notable exception, where the node assortativity distribution changed qualitatively.

Our results demonstrate that a 'strong rich-club', where the majority of links that originate from hubs terminate in other hubs, is very rarely present in real world networks, whereas a 'weak rich-club', where the link density is merely higher among hubs compared to the entire network, is present in many real-world networks. We note that a high node-to-link ratio is probably needed for the presence of a strong rich club. If such a rich club is present, it can indicate that the network is resilient to targeted attacks. However, assortative hubs may be rare in scale-free networks, due to the structural demands of a power-law degree distribution. In this paper we proposed an alternative approach to quantify node assortativity and applied it on a set of real world networks. It is our hope that our approach will be widely used to analyse a much larger set of real world networks in the future.

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